MATLAB Marina: Numerical Integration with Noisy Data

Numerical Integration with Noisy Data

One must be careful when performing numerical operations on noisy data (data with uncertainties or data with measurement error). The noise or error associated with measured or uncertain data is often small in magnitude but rapidly changing.

Integration tends to reduce the effect of error or noise whereas differentiation tends to magnify the effect of error or noise. Integration is similar to averaging and tends to reduce the effects of things that are small but vary rapidly.

Figure 1 shows a MATLAB program that generates a noisy signal and determines the numerical integral. The underlying function is a unit amplitude cosine of 250Hz. The additive noise is cosines of 60Hz and 120Hz with amplitudes uniformly distributed over the range -0.05 to 0.05 and -0.03 to 0.03. Figures 2a and 2b show plots of the noisy data and the approximate integral of the noisy data.

```
% 250Hz cosine with additive noise modeled as 60Hz and 120Hz
% cosines with varying amplitude
T = 1/250;
t = 0:T/20:5*T;
% unit amplitude cosine of 250Hz
g = 1.0*cos(2*pi*250*t);
% noise, random amplitude 60Hz and 120Hz cosines
noise60=(-0.05 + 0.1*rand(1,length(t))).*cos(120*pi*t);
noise120=(-0.03 + 0.06*rand(1,length(t))).*cos(240*pi*t);
% data plus noise
gNoisy = g + noise60 + noise120;

% approximate integral of the noisy data
intgNoisy = cumtrapz(t,gNoisy);
```

Figure 1. Program for Numerical Integral of Noisy Data

The noise in the data (Figure 2a) is most noticeable at the peaks. It is barely discernable in between the positive and negative peaks.

The integral of the noisy signal, shown in Figure 2b, is approximately a sine of 250Hz. The amplitude is much smaller than one might expect, but recall that $\int \cos(\omega t)dt = \frac{1}{\omega}\sin(\omega t) + c$, so the amplitude of the sine wave should be scaled by $\frac{1}{\omega}$ times the amplitude of the noisy signal. The noise is barely noticeable even at the peaks in the numerically integrated signal.

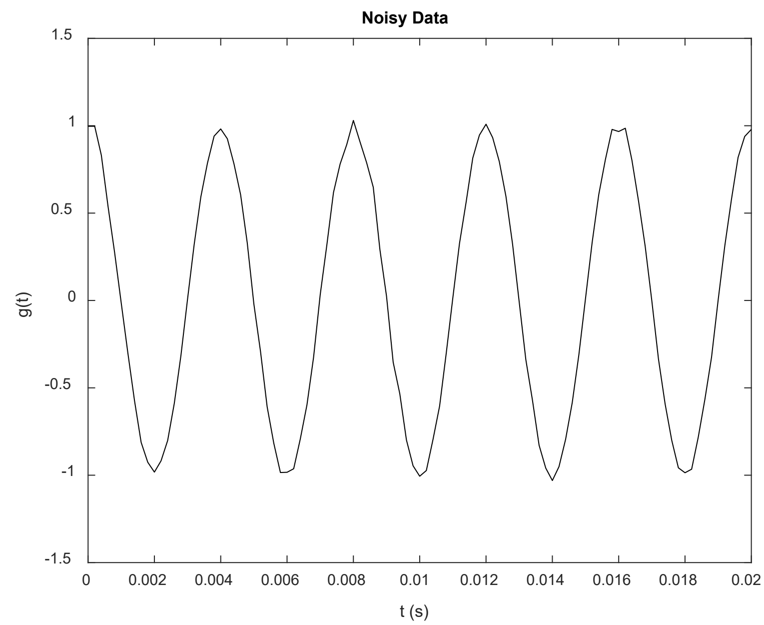

Figure 2a. Noisy Data

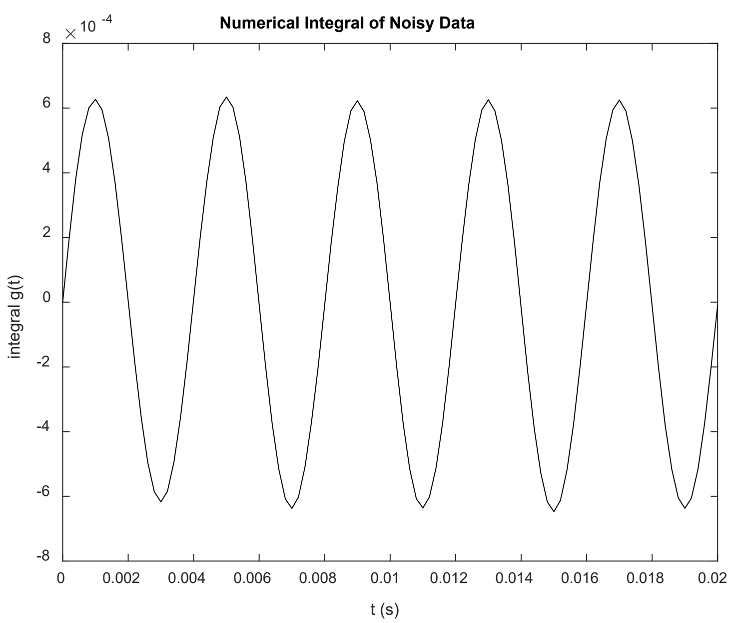

Figure 2b. Numerical Integral of Noisy Data

In this example the magnitude of the noise was 8% (5% plus 3%) or less of the original signal. The magnitudes were multiplied by a cosine of 60Hz or cosine of 120Hz which for most points reduced the magnitude of the noise. The average noise magnitude for the signal is around 2.3%.

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